Some Definitions of OPERATIONS RESEARCH:

Churchman,Ackoff and Arnoff: Operations research, in the most general sense, can be characterized as the application of scientific methods, techniques and tools, to problems involving the operations of a system so as to provide those in control of the operations with optimum solutions to the problems.

H. A. Taha: This new decision – making field has been characterized by the use of scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources.

Linear Programming

Linear programming is a mathematical modeling technique useful for economic allocation of scarce or limited resources, such as labor, material, machine, time, warehouse space, capital, energy, etc., to several competing activities such as products, services, jobs new equipment, projects, etc. on the basis of a given criterion of optimality.
In LPP the word linear refers to linear relationship among variables in a model. Thus, a given change in one variable will always causes a resulting proportional change in another variable. The word programming refers to modeling and solving a problem mathematically that involves the economic allocation of limited resources by choosing a particular course of action or strategy among various alternative strategies to achieve the desired objective.

Advantages of Linear Programming

1. Linear programming helps in attaining the optimum use of productive resources.
2. Linear programming techniques improve the quality of decisions.
3. Linear programming techniques provide possible and practical solutions since there might be other constraints operating outside the problem which must be taken into account.
4. Highlighting of bottlenecks in the production processes is the most significant advantage of this technique.
5. Linear programming also helps in re – evaluation of a basic plan for changing conditions.

Limitations of Linear programming

1. Linear programming treats all relationships among decision variables as linear. Neither the objective functions nor the constraints in real – life situations concerning business and industrial problems are linearly related to the variables.
2. While solving an LP model, there is no guarantee that we will get integer valued solutions. For example, in finding out how many men and machines would be required to perform a particular job a non – integer valued solution will be meaningless. Rounding off the solution to the nearest integer will not yield an optimal solution.
3. Linear programming model does not take into consideration of the effect of time and uncertainity.

ASSUMPTION OF LINEAR PROGRAMMING PROBLEM

Certainity: In all LP models, it is assumed, that all model parameters such as availability of resources, profit (or cost) contribution of a unit of decision variable and consumption of resources by a unit of decision variable must be known and may be constant.

Divisibility: The solution values of decision variables are allowed to assume continuous (fractional) values.

Additivity: The value of objective function and the total amount of resource used (or supplied), must be equal to the sum of the respective individual contributions (profit or cost) by decision variables. For example, the total profit earned from the sale of two products A and B must be equal to the sum of the profits earned separately from A and B.

Linearity: The amount of each resource used (or supplied) and its contribution to the profit(or cost) in objective function must be proportional to the value of each decision variable. For example, if production of one unit of a product uses 5 hours of a particular resource, then making 3 units of that product uses 3 x 5 = 15 hours of that resource.

Definitions

(i) Solution: Values of decision variables x_j (j=1,2,... ,n) which satisfy the constraints of a general L. P. problem is called the solution to that L.P. problem.

(ii) Feasible Solution: Any solution that also satisfies the non-negative restrictions of the general L.P. problem is called a feasible solution.

(iii) Optimal Solution: Any basic feasible solution which optimizes (minimizes or maximizes) the objective function of a general L. P. problem is called an optimal basic feasible solution to the general L.P. problem.

GRAPHICAL METHOD
(EXTREME POINT METHOD)

The graphical solution procedure is one method of solving two variable linear programming problems and involves the following steps:

1. Formulate the problem in terms of a series of mathematical constraints and an objective function.

2. Plot each of the constraints as follows: Each inequality in the constraint equation be written as equality. Give any arbitrary value(preferably choose 0) to one variable and get the value of other variable by solving the equation. Similarly, give another arbitrary value(preferably choose 0) to the variable and find the corresponding value of the other variable. Now plot these two sets of values. Connect these points by a straight line. This exercise is to be carried out for each of the constraint equations. Thus, there will be as many straight lines as there are equations; each straight line representing one constraint.

3. Identify the feasible region (or solution space), i.e., the area which satisfies all the constraints simultaneously. For 'greater than' constraints, the feasible region will be the area

which lies above the constraint lines. For 'less than' constraints, this area is generally the region below these lines. For 'greater than or equal to' or 'less than or equal to' constraints, the feasible region includes the points on the constraint lines also.

4. Identify each of the corner(or extreme points) of the feasible region either by visual inspection or the method of simultaneous equations.

5. Compute the profit/cost at each corner point by substituting the co-ordinates of that point into the objective function.

6. Identify the optimal solution at that corner point which shows highest profit (in a maximization problem) or lowest cost (in a minimization problem).

NORTH – WEST CORNER METHOD

STEP – 1 Select the north-west corner cell of the transportation table and allocate ??? ? ? ?? ???????.

STEP – 2 If $b_1 > a_1$, we move vertically to the second row and make the second allocation of magnitude ??? ? ? ?? ?????? ???? in the cell (2,1).

If $b_1 < a_1$, we move right horizontally to the second column and make the second allocation of magnitude ??? ? ? ????? ??? ???? in the cell (1,2).

If $b_1 = a_1$, there is a tie for the second allocation. One can make the second allocation of magnitude ??? ? ? ?? ?????? ??? = 0 in the cell(1,2)

??? ? ? ????? ?? ???? = 0 in the cell (2,1).

STEP -3. Repeat steps 1 and 2 moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

LEAST COST METHOD (MATRIX MINIMA METHOD)

STEP – 1. Determine the smallest cost in the cost matrix of the transportation table. Let it be ???.

Allocate ??? ? ? ?? ???????in the cell (I, j).

STEP -2.If $x_{ij} = a_i$ cross off the i th row of the transportation table and decrease b_j by a_i. Go to step 3.

If $x_{ij} = b_j$ cross off the j th column of the transportation table and decrease a_i by b_j. Go to step 3.

If $x_{ij} = a_i = b_j$ cross off either the i th row or j th column but not both.

STEP – 3. Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

VOGEL APPROXIMATION METHOD

STEP -1.For each row of the transportation table identify the smallest and the next-to-smallest costs. Determine the difference between them for each row and write it in Row penalty column. Similarly, compute the differences for each column and write it in column penalty row.

STEP -2.Identify the row or column with the largest penalty among all the rows and columns. If a tie occurs, use any arbitrary tie-breaking choice. Let the greatest penalty correspond to i th row and let c_{ij} be the smallest cost in the i th row. Allocate ??? ? ? ?? ??????? in the (i, j) th cell and cross off the i th row or the j th column in the usual manner.

STEP-3. Recompute the column and row penalty for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

Hungerian Method

An efficient method for solving an assignment problem as developed by Hungerian mathematicians D. Konig is summarized below:

Step 1.Check whether the number of rows is equal to number of columns; if not then add dummy rows/columns to make square matrix. The cost entries of dummy rows/columns are taken as zero.

Step 2. Find the smallest element in each row of the given cost matrix ad then subtract this minimum from all the elements of that row.

Step 3. Find the smallest element in each column of the given cost matrix ad then subtract this minimum from all the elements of that column. Each row and column now have atleast one zero.

Step 4. In the modified obtained in step 3, search for an optimal assignment as follows:

(a) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (▭) and cross off (x) all other zeros in its column. Continue in this manner until all the rows have been taken care of.

(b) Repeat the procedure for each column of the reduced matrix.

(c) If a row and/or column have two or more zeros and one cannot be chosen by inspection then assign arbitrary any one of these zeros and cross off all other zeros of that row/column.

(d) Repeat (a) through (c) above successively until the chain of assigning (▭) or cross (x) ends.

Step 5. If the number of assignments (▭) is equal to n, an optimum solution is reached. If the number of assignments is less than n (the order of the matrix), go to the next step.

Step 6. Draw the minimum number of horizontal and/or vertical lines to cover all the zeros of the reduced matrix.

Step 7. Develop the new revised cost matrix as follows:

(a) Find the smallest element of the reduced matrix not covered by any of the lines.

(b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines. The elements which are covered by the lines remain unchanged.

Step 8. Go to Step 5 and repeat the procedure until an optimum solution is attained.

REPLACEMENT PROBLEM

Replacement theory is concerned with the problem of replacement of machines, electricity bulbs, men etc. due to their deteriorating efficiency, failure or breakdown. Replacement is carried out under the following situations: When existing items have outlived their effective lives and it may not be economical to continue with them anymore; and Items which might have been destroyed either by accident or otherwise.

The above replacement situations may be categorized into the following four categories:

- Replacement of items that deteriorates with time e.g. machine tools, vehicles, equipment, buildings, etc.
- Replacement of items which do not deteriorate but fail completely after certain amount of use, ex electric bulbs, T. V. parts, etc.
- Replacement of an equipment (or item) that becomes out of date due to new developments.
- The existing working staff in an organization gradually diminishes due to death, retirement and other reasons.

Project Management

A project is defined as a collection of interrelated activities with each activity consuming time and resources.

Examples of Project,
Construction of a bridge, Construction of Highway, Power plant, House, design, development and marketing of a new product, research and development of a work.

Such projects involve large number of interrelated activities which must be completed in a specified time, in a specified sequence and require resources such as personnel, money, materials, facilities and/or space.

Objective of Project Management

Schedule the required activities in an efficient manner so as to complete it on or before a specified time limit at a minimum cost of its completion.

Events: Events in the network diagram represent project milestones, such as the start or the completion of an activity (task) or activities, and occur at a particular instant of time at which some specific part of the project has been or is to be achieved. Events are commonly represented by circles (nodes) in the network diagram.

Activities: Activities in the network diagram represent project operations or tasks to be conducted. As such activities except dummy consume time and resources and incur costs. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project.

Predecessor Activity: An activity which must be completed before one or more other activities start is known as predecessor activity.

Successor Activity: An activity which started immediately after one or more of other activities are completed is known as successor activity.

Dummy Activity: An activity which does not consume either any resource and/or time is known as dummy activity.
Errors in Network

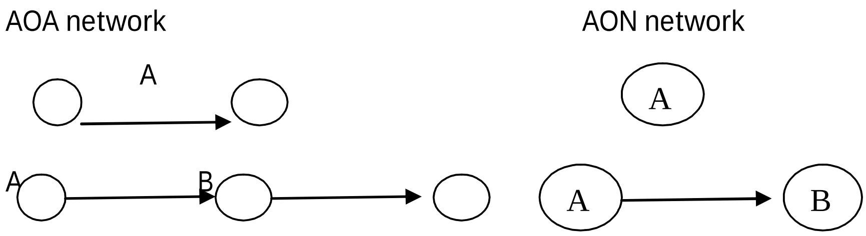
Activity-on-Node (AON) Network: In this type of precedence network each node (or circle) represents a specific task while the arcs represent the ordering between tasks. AON network diagrams place the activities within the nodes, and the arrows are used to indicate sequencing requirements. Generally, these diagrams have no particular starting and ending nodes for the whole project. The lack of dummy activities in these diagrams always makes them easier to draw and to interpret.

Activity-on-Arrow (AOA) network: In this type of precedence network at each end of the activity arrow is a node (or circle). These nodes represent points in time or instants, when an activity is starting or ending. The arrow itself represents the passage of time required for that activity to be performed.

These diagrams have a single beginning node from which all activities with no predecessors may start. The diagram then works its way from left to right, ending with a single ending node, where all activities with no followers come together. Three important advantages of using AOA are as follows:

(i) Many computer programs are based on AOA network.
(ii) AOA diagrams can be superimposed on a time scale with the arrows drawn, the correct length to indicate the time requirement.
(iii) AOA diagrams give a better sense of the flow of time throughout a project.

1. Activity A
2. B must follow A



Rules of network Construction

- Arrows represent activities and circles the events.
- Each activity should be represented by only one arrow and must start and end in a circle called event.
- The event numbered 1 denotes start of the project and is called initial event. Events carrying highest number denote the completion event.
- The number at an activity's head should always be larger than that at its tail. (i<j)

. An activity must be uniquely identified by its starting and completion event which implies that...

i) An event number should not get repeated.

ii) Two activities should not be identified by the same completion events.

iii) Activities must be represented either by their symbols or by the corresponding ordered pair of starting-completion events.

- The logical sequence between activities must follow the following rules.

i) An event cannot occur until all the incoming activities into it have been completed.

ii) An activity cannot start unless all the preceding activities, on which it depends, have been completed.

iii) Though a dummy activity does not consume either any resource or time, even then it has to follow all the rules.

Critical Path: The critical activities of a network that constitute an uninterrupted path which spans the entire network from start to finish is known as critical path.

The critical path is the sequence of critical activities that form a continuous path between the start of a project and its completion. This is critical in the sense that if any activity in this sequence is delayed, the completion of the entire project will be delayed. The critical path is shown by a thick line or double lines in the network diagram.

The length of the critical path is the sum of the individual times of all the critical activities lying on it and defines the longest time to complete the project.

Float(Slack) of an Activity and Event

The *float (slack)* or *free* time is the length of time in which a non-critical activity and/or an event can be delayed or extended without delaying the total project completion time.

Float(Slack) of an Event: The Slack(s) also called float of an event is the difference between its latest occurrence time (L_i) and its earliest occurrence time (E_i). That is:

$$\text{Event Float} = L_i - E_i$$

It is a measure of how long an event can be delayed without increasing the project completion time.

PERT ALGORITHM

Let t_o = The optimistic time, is the shortest possible time to complete the activity if all goes well.

t_p = The pessimistic time, is the longest time that an activity could take if every thing goes wrong.

t_m = The most likely time, is the estimate of the normal time an activity would take. If only one time were available, this would be it.

Step -1 Compute t_e $\frac{t_o + 4t_m + t_p}{6}$ and σ_i^2 $\left(\frac{t_p - t_o}{6}\right)^2$ for each activity.

Difference between PERT AND CPM

PERT	*CPM*
A Probability model with uncertainty in activity duration. There are three time estimates namely optimistic time, pessimistic time and most likely time.	A deterministic model with well-known activity (single) times based upon the past experience.
It is said to be event oriented.	It is activity oriented.
PERT is generally used for those projects where time required to complete various activities is not known a priori.	CPM is commonly used for those projects which are repetitive in nature and where one has prior experience of handling similar projects.

ADVANTAGES OF PERT

1. Forces management to plan carefully and study how the various parts fit into the whole project.
2. Focuses attention on the critical elements of the project. It helps the management to take selective attention for expediting activity to maintain the schedule.
3. Provides up-to-date status information through frequent reporting, data processing and accurate programme analysis.
4. Helps to formulate new schedules when the existing schedule cannot be met, and serve the planning and evaluation at all levels.

5. Minimizes production delays, interruptions and conflicts by scheduling and budgeting resources.

DRAWBACKS OF PERT

1. Takes only time into consideration and not the cost.
2. Time estimates to perform activities constitute a major limitation of this technique.
3. Calculation of probabilities under PERT approach is done on the assumption that a large number of independent activities operate on critical path and as such the distribution of total time is normal which may not be true in real life situations.
4. Out of three time estimates, the pessimistic time varies widely from the mean and is always biased.
5. Any change in the technological parameters involves reworking of the entire network. In many projects it is not possible to estimate time and technological dependencies.

ADVANTAGES OF CPM

1. Helps the top management to concentrate their attention to the critical activities and their completion in time.
2. Provides the knowledge of critical and non-critical activities. This helps the management to divert the resources from non-critical activities to the critical activities.
3. Provides a best way of planning and scheduling a large construction project.
4. Gives the complete information about the importance, duration, size and performance of an activity.

DRAWBACKS OF CPM

1. CPM assumes deterministic time approach which may not be possible in all real life situations.
2. It is difficult to use CPM as controlling device because if there are certain changes in activity duration then whole calculation has to be repeated again and new critical path would be found.
3. Does not incorporate statistical analysis in determining the time estimates.